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Portfolio optimization using the GO-GARCH model: evidence from Ukrainian Stock Exchange

Abstract. This paper provides an experimental study on optimal portfolio composition. Data on seven stocks, included in Ukrainian Exchange Index, for the period from January to December 2015 are considered. In total, seven big industrial, electric and military companies are selected from the Ukrainian Exchange: Avdiivka Coke Plant, PJSC; Azovstal Iron and Steel Works, PJSC; Raiffeisen Bank Aval, JSC; Centerenergo, PJSC; Donbasenergo, PJSC; Motor Sich, JSC; and Ukrnafta, OPJC. The sample amounts to 226 observations.

The analysis covers descriptive statistics, correlation, and, finally, optimal investment weights, which are calculated using Sharpe ratio. Covariance matrix of returns is estimated by means of generalized orthogonal GARCH model with Gaussian and normal-inverse Gaussian distributions for errors. Selected stocks during the considered period have on average negative rates of returns. At the same time, these stocks in most of cases are positively correlated with each other, leading hence to a fewer room for the efficient diversification. Both Gaussian and normal-inverse Gaussian portfolios preclude that on average investment weights one should be focused mainly on Centerenergo and Motor Sich stocks. Based on these results, the investor should buy 46% of Centerenergo's stocks and 34% of Motor Sich's stocks.

Selected stocks during the considered period have on average negative rates of returns. At the same time, these stocks in most of cases are positively correlated with each other, leading hence to a fewer room for the efficient diversification.

Despite this, our results denoted that implementation of multivariate GARCH together with normal-inverse Gaussian distribution for errors enables to reduce the portfolio risk substantially. Comparing optimal GO-GARCH portfolios with naïve portfolio with all weights equal and Ukrainian Exchange Index we demonstrate that the former provide smaller portfolio variance and better VaR than naïve portfolio and the Index.

Keywords: Portfolio; GO-GARCH Model; Return; Risk; Optimizations; Stock Exchange; Portfolio
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Оптимізація інвестиційного портфеля за допомогою моделі GO-GARCH (на матеріалах Української фондової біржі)

Анотація. У статті представлено емпіричне дослідження процесу формування оптимального портфеля на основі даних семи акцій, які включено до індексу Української біржі, за період з січня по грудень 2015 року.

Аналіз включає в себе описову статистику, кореляцію та розрахунок оптимальних ваг у портфелі. Варіаційно-коваріаційна матриця дохідностей оцінена за допомогою ортогональної моделі GO-GARCH з гаусовим і нормальним-зворотним гаусовим розподілами для залишків.

Результати показують, що застосування багатовимірної GARCH моделі з нормальним-зворотним гаусовим розподілом для залишків дозволяє істотно знизити портфельний ризик. Порівнюючи GO-GARCH портфелі з наївним портфелем, в якому ваги всіх активів рівні, і індексом Української біржі, ми продемонстрували, що перші забезпечують меншу дисперсію портфеля і менший VaR, ніж наївний портфель та індекс.

Ключові слова: портфель; модель GO-GARCH; дохідність; ризик; оптимізація.

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Оптимизация инвестиционного портфеля с помощью модели GO-GARCH (на материалах Украинской фондовой биржи)

Аннотация. В данной работе проведено эмпирическое исследование процесса формирования оптимального портфеля на основе данных по семи акциям, входящим в индекс Украинской биржи, за период с января по декабрь 2015 года. Анализ включает в себя описательную статистику, корреляцию и расчет оптимальных весов в портфеле, основываясь на применении коэффициента Шарпа. Вариационно-ковариационная матрица доходностей оценена с помощью обобщённой ортогональной GARCH модели с гауссовым и нормальным-обратным гауссовым распределениями для остатков. Результаты исследования показывают, что применение многомерной GARCH модели с нормальным-обратным гауссовым распределением для остатков позволяет существенно снизить портфельный риск. Сравнивая GO-GARCH портфели с наивным портфелем, в котором веса всех активов равны, и индексом Украинской биржи, мы продемонстрировали, что первые обеспечивают меньшую дисперсию портфеля и меньший VaR, чем наивный портфель и индекс.

Ключевые слова: портфель; модель GO-GARCH; доходность; риск; оптимизация.

1. Introduction

Financial intermediaries are essential participants in the investment process. They play an important role in the investment market, acting as intermediaries in the accumulation and redistribution of temporarily free funds. To perform the mission of capital protection and enhancement, financial intermediaries should constantly improve the efficiency of their activities in the securities market and work on improvement of analytical instruments used in management process.

Risk exposure prevention requires intense diversification of the investment portfolio. Improving the asset allocation efficiency for financial intermediaries through diversification can not only significantly reduce the investment risk, the probability and amount of loss on the stock market, but also create conditions for improved financial results.

2. Brief Literature Review

Theoretical and practical aspects of the portfolio investment through diversification, in particular solving problems related to finding an optimal balance of assets in the portfolio, as well as the calculation of their cost, were studied by many prominent scholars, including several Nobe Prize winners in Economics: Harry Markowitz (Markowitz, 1952) [1], William Sharpe (Sharpe, 1964) [2], James Tobin (Tobin, 1985) [3] and others. They developed a theory of the investment portfolio, conducted a mathematical study of criterion «risk-return» and described the construction of optimal portfolio weights. The investment portfolio theory is based on the mean-variance efficiency for assets allocation, pioneered by Harry Markowitz (Markowitz, 1952, 1959) [1, 4] and further developed by William Sharpe (Sharpe, 1963) [2]. Moreover, Capital Asset Pricing Model (CAPM) was developed by William Sharpe (Sharpe, 1964) [5], Joan Lintner (Lintner, 1965) [6] and Jan Mossin (Mossin, 1966) [7]. Arbitrage pricing theory was pioneered by Stephen Ross (Ross, 1976) [8].

Philosophy of index investing originated in the early 1950s, when John Bogle, Princeton University graduate, in his Master's thesis showed that two-thirds of mutual funds provided their shareholders, through the implementation of active investment strategies that existed at that time, with yield which was not greater than if they had just carried out investments in shares of companies following the structure of a generally known stock index. Eventually, in 1976 John Bogle founded the first index fund for individual investors, now called Vanguard 500 Index Fund, which had investments in stocks included in the S&P 500 index by buying securities in amounts correlated to weighting factor of the shares in the index. Other indices primarily used for investment are Dow Jones, Russell, and NASDAQ.

Today scientists actively investigate the issue of rational behavior of investors in the securities market in the process of investment portfolio optimization, study how investors make decisions and how they forecast the price of securities. Overall, among the latest contributions to the subject we would like to note the following scholars.

Danielsson, J., Jorgensen, B. N., de Vries, C. G. (Danielsson, Jorgensen, de Vries, 2008) [9] characterized the investor's optimal portfolio allocation subject to a budget constraint and a probabilistic VaR constraint in complete markets environments with a finite number of states. Fernandes, B., Street, A., Valladao, D., Fernandes, C. (Fernandes, Street, Valladao, Fernandes, 2016) [10] provided a new perspective on robust portfolio optimization where they imposed an intuitive loss constraint for the optimal portfolio considering asset returns in a data-driven polyhedral uncertainty set. To tackle mean-VaR portfolio optimization within the actual portfolio framework (APF), Rankovic, V., Drenovak, M., Urosevic, B., Jelic, R. (Rankovic, Drenovak, Urosevic, Jelic, 2016) [11] proposed a novel mean-VaR optimization method where VaR is estimated using a univariate Generalized AutoRegressive Conditional Heteroscedasticity (GARCH) volatility model. The optimization was performed by employing a Nondominated Sorting Genetic Algorithm (NSGA-II). Mei, X., DeMiguel, V., Nogales, F. J. (Mei, DeMiguel, Nogales, 2016) [12] analyzed the optimal portfolio policy for a multi-period mean-variance investor facing multiple risky assets in the presence of general transaction costs. For proportional transaction costs, they gave a closed-form expression for a no-trade region, shaped as a multi-dimensional parallelogram, and showed how the optimal portfolio policy can be efficiently computed for many risky assets by solving a single quadratic program. Luo, C., Seco, L., Bill Wu L.-L. (Luo, Seco, Bill, 2015) [13] investigated and compared performances of the optimal portfolio selected by using the Orthogonal GARCH (OGARCH) Model, Markov Switching Model and the Exponentially Weighted Moving Average (EWMA) Model in a fund of hedge funds. Vercher, E., Bermudez D. J. (Vercher, Bermudez, 2015) [14] introduced a cardinality constrained multi-objective optimization problem for generating efficient portfolios within a fuzzy mean-absolute deviation framework. They assumed that the return on a given portfolio was modeled by means of LR-type fuzzy variables, whose credibility distributions collect the contemporary relationships among the returns on individual assets. Using daily returns of the S&P 500 stocks from 2001 to 2011, Mainik, G., Mitov, G., Ruschendorf, L. (Mainik, Mitov, Ruschendorf, 2015) [15] performed a backtesting study of the portfolio optimization strategy based on the Extreme Risk Index (ERI). This method used multivariate extreme value theory to minimize the probability of large portfolio losses.

Despite the presence of a number of studies dedicated to «portfolio optimization», the analysis for the Ukrainian Stock Exchange was not conducted, so we decided to fill this gap.

3. The purpose of the article is to use a mathematical model to minimize risk for a given portfolio. In this study we examine the portfolio consisted of seven stocks, included in

Ukrainian Exchange Index (on June 16, 2016 list of the UX Index constituent stocks changed, and now it contains only five stocks [16]). We calculate the optimal weights using Sharpe ratio (Sharpe, 1966) [17]. The estimation of portfolio assets' conditional covariance is conducted by means of generalized orthogonal GARCH model (Van der Weide, R., 2002) [18] with multivariate normal and normal-inverse Gaussian distributions for errors. Comparing optimal portfolios with naïve portfolio with all weights equal we demonstrate that implementation of multivariate GARCH together with normal-inverse Gaussian distribution for errors enables to reduce the portfolio risk substantially.

4. Methodology

Firstly, we have n financial time series of length T:

$$x_t = \left(x_{1t}, \ldots, x_{nt}\right)', \ t = 1, \ldots, T \tag{1}$$

x_t are observable returns, which are demeaned by vector autoregression model with intercept:

$$y_t = x_t - \mathrm{E}\left(x_t \mid \mathrm{F}_{t-1}\right), \tag{2}$$

$$\mathrm{E}\left(x_t \mid \mathrm{F}_{t-1}\right) = \mathrm{A} + \mathrm{B}x_{t-1} \equiv E, \tag{3}$$

where A and B are matrices of parameters. The resulted variable y_t is usually called innovations. Innovations are used in GARCH-type models to estimate volatility Σ_t:

$$y_t = \Sigma_t^{1/2}\varepsilon_t, \varepsilon_t \sim F(\theta), \tag{4}$$

$$\Sigma_t = \mathrm{E}(y_t y_t' \mid \mathrm{F}_{t-1}). \tag{5}$$

Standardized innovations ε_t are distributed according to some known distribution F with parameter set θ.

The multivariate GARCH models are usually estimated by means of maximum likelihood method with log likelihood function:

$$L_t = -\frac{1}{2}\left(log\left|\Sigma_t\right| + y_t'\Sigma_t^{-1}y_t\right), \tag{6}$$

where means determinant.

We | | found optimal weights of the portfolio by maximizing the Sharpe ratio Z.

$$Z = \frac{\mu_p - r_f}{\sigma_p} = \frac{w'\mu - r_f}{\left(w'\Sigma w\right)^{1/2}}, \tag{7}$$

where μ_p - portfolio mean return,
r_f - risk-free rate,
σ_p - portfolio standard deviation,
w - portfolio weights,
μ - vector of assets' returns,
Σ - variance-covariance matrix of returns.

Evidently, the sum of weights w should be equal to 1. The well-known solution of this problem can be written as follows (see, for example, Zivot, 2011) [19]:

$$w^* = \frac{\Sigma^{-1}\left(\mu - r_f \cdot \vec{1}\right)}{\vec{1}'\Sigma^{-1}\left(\mu - r_f \cdot \vec{1}\right)}, \tag{8}$$

where $\vec{1}$ - vector of ones. Due to the fact that most of the assets demonstrate negative mean return (see Table 1) we allow short selling in our portfolio.

We allow to assume time-varying covariance matrix and model it via GO-GARCH model, which enables us to obtain dynamic conditional covariance and control for autocorrelation and heteroskedasticity in returns (Engle, Kroner, 1995) [20].

$$\Sigma_t = \mathrm{X}\mathrm{V}_t\mathrm{X}', \tag{9}$$

X is an $n \times n$ orthogonalization matrix, V_t - diagonal matrix with diagonal elements v_t which follow the equation:

$$v_t = c + D(y_{t-1} \odot y_{t-1}) + Kv_{t-1}, \tag{10}$$

where D and K - diagonal matrices of parameters, c - $n \times 1$ vector, $\odot$ - element-wise product. To ensure Σ_t matrix to be positive definite, elements of D, K and c should be positive.

In our paper we chose multivariate Gaussian and normal-inverse Gaussian distributions (see for example Feller, 2008 [21]) for standardized innovations ε_t.The first one is a parsimonious and heavily studied distribution with many useful properties.

$$f_G\left(\mathbf{x}\right) = (2\pi)^{-\frac{n}{2}} \mid \mathbf{\Sigma} \mid^{-\frac{1}{2}} e^{-\frac{1}{2}(\mathbf{x}-\boldsymbol{\mu})'\mathbf{\Sigma}^{-1}(\mathbf{x}-\boldsymbol{\mu})}, \tag{11}$$

The distribution takes into account only two first stochastic moments, represented by location parameter $\boldsymbol{\mu}$ and covariance matrix Σ . But there is a well-known fact from empirical finance, that the empirical distribution of returns tends to be skewed (see, e.g. Harvey, Siddique, 2000) [22]). So along with Gaussian distribution we implement normal-inverse Gaussian distribution for errors (Oigard et al., 2005) [23].

$$\begin{aligned} f_{NIG}(\mathbf{x}) &= \frac{\delta}{2^{(n-1)/2}}\left[\frac{\alpha}{\pi q(\mathbf{x})}\right]^{(n+1)/2} exp[\,p(\mathbf{x})]K_{(n+1)/2}[\alpha q(\mathbf{x})], \\ q(\mathbf{x}) &= \sqrt{\delta^2 + \left[\left(\mathbf{x}-\boldsymbol{\mu}\right)^T \mathbf{\Gamma}^{-1}\left(\mathbf{x}-\boldsymbol{\mu}\right)\right]}, \\ p(\mathbf{x}) &= \delta\sqrt{\alpha^2 - \boldsymbol{\beta}^T\mathbf{\Gamma}\boldsymbol{\beta}} + \boldsymbol{\beta}^T(\mathbf{x}-\boldsymbol{\mu}). \end{aligned} \tag{12}$$

$K_{(n+1)/2}$ is the modified Bessel function of the second kind with index $(n+1)/2$, $\alpha, \delta, \boldsymbol{\beta}, \boldsymbol{\mu}, \mathbf{\Gamma}$ are parameters. α controls the heaviness of the tails (smaller values of α implies heavier tails); $\boldsymbol{\beta}$ is a vector skewness parameter; δ is a scale parameter; $\boldsymbol{\mu}$ is location parameter; $\mathbf{\Gamma}$ account for correlation between assets. The flexibility of normal-inverse Gaussian distribution allows to take into account the skewness of returns along with the time-dependent volatility.

5. Empirical results

5. 1. Data

Daily data used in this study are downloaded from [24]. The period under consideration lasts from January 6, 2015 till December 30, 2015. Totally, seven firms are selected from the Ukrainian Exchange:

- Avdiivka Coke Plant, PJSC, Common (AVDK),
- Azovstal Iron and Steel Works, PJSC, Common (AZST),
- Raiffeisen Bank Aval, JSC, Common (BAVL),
- Centerenergo, PJSC, Common (CEEN),
- Donbasenergo, PJSC, Common (DOEN),
- Motor Sich, JSC, Common (MSICH), and
- Ukrnafta, OPJC, Common (UNAF).

The sample amounts to 226 observations. Table 1 presents descriptive statistics for the selected stocks' logarithmic returns.

According to Table 1, UNAF, BAVL and MSICH demonstrate higher average rate of return, and, at the same time, moderate risk, estimated by standard error. On the other hand, stocks with low average returns, such as AVDK, AZST and DOEN have higher risk. CEEN is positioned between these two subgroups with relatively low average return and small risk.

Based on the fundamental relationship between risk and return, we expect that stocks which demonstrate higher return and lower risk are to be invested with positive weights. As this relation will be stronger, proportions of invested funds at these stocks will increase. Vice versa, as this relationship will be weakened, proportions of invested funds should decrease.

Table 2 presents correlation coefficients of returns.

Higher correlation coefficient is found between CEEN and UNAF, i.e. 0.48. In overall, more than 70% of correlation coefficients between returns are positive, what leaves little room for further diversification. As suggested in Damodaran (1996) [25], for the risk free rate we choose the rate of long-term Ukrainian government bonds with expiration on 2026, which is 7.75% [26].

Tab. 1: **Descriptive statistics**

	AVDK	AZST	BAVL	CEEN	DOEN	MSICH	UNAF
Mean	-0.0022	-0.0026	-0.0010	-0.0021	-0.0031	-0.0010	-0.0007
Standard Error	0.0029	0.0020	0.0014	0.0011	0.0016	0.0009	0.0017
Median	-0.0007	-0.0013	-0.0011	-0.0015	-0.0004	-0.0004	-0.0010
Mode	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	-0.0144
Standard Deviation	0.0442	0.0314	0.0211	0.0167	0.0250	0.0140	0.0260
Sample Variance	0.0020	0.0010	0.0004	0.0003	0.0006	0.0002	0.0007
Kurtosis	16.2246	6.6425	0.8107	12.9403	2.5899	4.1995	5.3970
Skewness	1.5407	-0.7623	0.0001	-1.6326	-0.5769	-0.5876	0.6972
Range	0.5010	0.3279	0.1275	0.1756	0.1936	0.1207	0.2496
Minimum	-0.1622	-0.1920	-0.0642	-0.1261	-0.1054	-0.0722	-0.0991
Maximum	0.3389	0.1359	0.0633	0.0495	0.0882	0.0485	0.1504
Sum	-0.5021	-0.6392	-0.2408	-0.5144	-0.7381	-0.2448	-0.1595

Source: Compiled by the authors

Tab. 2: **Correlation coefficients**

	AVDK	AZST	BAVL	CEEN	DOEN	MSICH	UNAF
AVDK	1.0000						
AZST	-0.0010	1.0000					
BAVL	0.0803	0.0173	1.0000				
CEEN	0.0357	0.3382	-0.0344	1.0000			
DOEN	-0.0474	0.2131	-0.0558	0.3190	1.0000		
MSICH	-0.0800	0.0518	0.0570	0.1575	0.1166	1.0000	
UNAF	0.0390	0.3315	-0.0547	0.4798	0.3021	0.1712	1.0000

Source: Compiled by the authors

Obviously, both Gaussian and normal-inverse Gaussian portfolios provide smaller portfolio variance than raw assets and naïve portfolio. Moreover the tails of portfolio distributions are much lighter for the estimated portfolios.

Table 4 presents the estimation of portfolio risk via portfolio returns' standard deviation and VaR.

Here we choose 95% level of confidence for VaR as suggested in Riskmetrics [27]. Naïve portfolio demonstrates the poorest results. Index portfolio performs slightly better. The use of GO-GARCH allows reducing substantially the risk of portfolio estimated by standard deviation and VaR.

We also compare average returns of GO-GARCH portfolios, naïve portfolio and index in Table 5. The returns are multiplied by 1000 for the sake of convenience.

GO-GARCH portfolios provide better returns, than naïve and index portfolios. Moreover, they allow investors to take lower risk (see Table 4, Figure 1 and Figure 2).

5. 2. Portfolio weights

Table 3 contains the average weights of all the assets for two selected probability distributions.

Both Gaussian and normal-inverse Gaussian portfolios preclude that on average investment weights one should be focused mainly on Centerenergo (CEEN) and Motor Sich (MSICH). Based on these results, the investor should buy 46% of Centerenergo's stocks and 34% of Motor Sich's stocks.

For illustrative purpose we present kernel estimates of portfolio returns with normal errors and of raw assets' returns on Figure 1 and Figure 2. We also add a naïve portfolio with equal weights and the returns of UX index on the figures.

Tab. 4: **Portfolio risk estimation**

	sd	VaR
norm	0.0114	-0.0252
nig	0.0119	-0.0252
UX	0.0149	-0.0262
naive	0.0153	-0.0274

Note:
«norm» stands for GO-GARCH portfolio with Gaussian errors,
«nig» stands for GO-GARCH portfolio with normal-inverse Gaussian errors,
«sd» stands for portfolio returns' standard deviation,
«VaR» stands for Value-at-Risk.
Source: Elaborated by the authors

Tab. 3: **Average weights**

	AVDK	AZST	BAVL	CEEN	DOEN	MSICH	UNAF
Gaussian	-0.0765	-0.0043	0.1076	0.4608	0.1589	0.3430	0.0106
Normal -inverse Gaussian	-0.0541	0.0142	0.1058	0.4418	0.1543	0.3286	0.0094

Source: Compiled by the authors

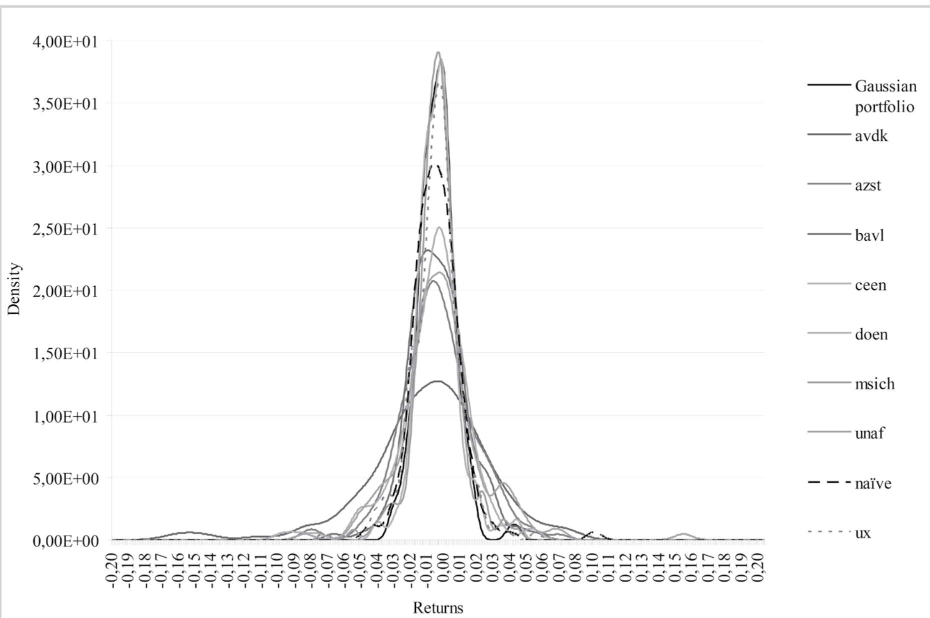

Fig. 1: **Kernel estimates of returns in comparison with Gaussian, naïve portfolio and UX index**
Source: Elaborated by the authors

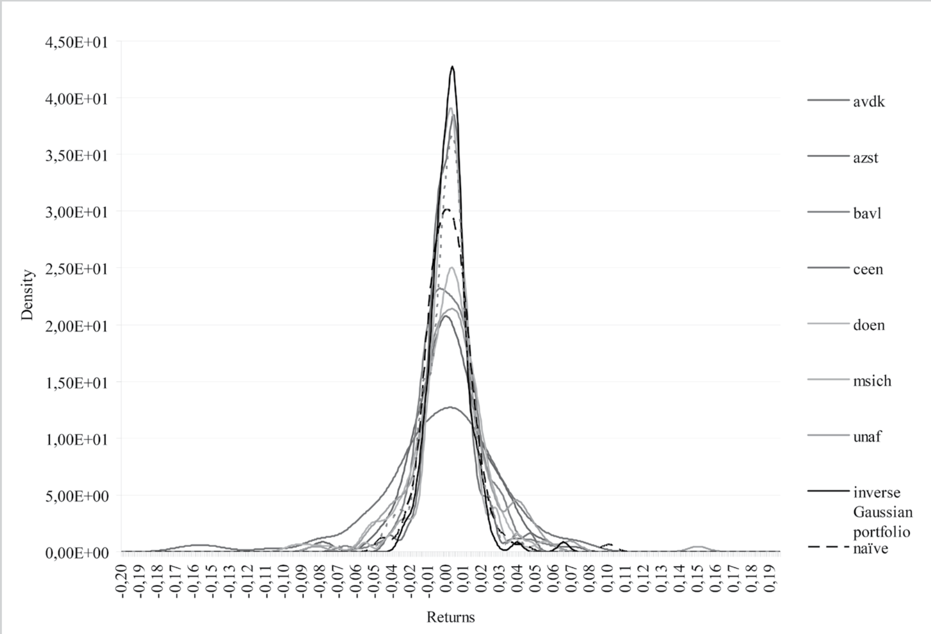

Fig. 2: **Kernel estimates of returns in comparison with normal-inverse Gaussian, naïve portfolio and UX index**
Source: Elaborated by the authors

Tab. 5: **Average returns of the portfolios**

	Average return*1000
norm	-0.600
nig	-0.541
UX	-1.678
naive	-0.851

Note: The returns are multiplied by 1000
Source: Elaborated by the authors

6. Conclusion

The purpose of this study was to present a methodological and an empirical approach to portfolio optimization. We used dynamic covariance to calculate optimal weights. The dynamic covariances are estimated via GO-GARCH model with Gaussian and normal-inverse Gaussian innovations. The comparison of risk estimated by 95% VaR and returns' standard deviation showed that GO-GARCH portfolios outperform both naïve and index portfolios. Moreover GO-GARCH with normal-inverse Gaussian errors results in portfolio with considerably thinner tails due to the fact that this distribution enables to capture returns' heavy tails (see Figure 1 and Figure 2).

As for the returns GO-GARCH with both distributions ensure higher average returns.

The best performance is demonstrated by normal-inverse Gaussian portfolio with the highest average return and the risk, which is the same according to VaR and slightly higher according to standard deviation as in Gaussian GO-GARCH. To sum up, the use of normal-inverse Gaussian errors, which implement skewness in modeling volatility, allow increasing the performance of asset allocation process and surpass both naïve and index portfolio.

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